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**MELBOURNE, VICTORIA**

Aircraft Systems Technical Memorandum 98

**FPG2 - A FLIGHT PROFILE GENERATOR PROGRAM (U)**

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SEP 18 1989  
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by  
R.B. MILLER

Approved for public release

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**FPG2 - A FLIGHT PROFILE GENERATOR PROGRAM (U)**

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R.B. MILLER

**SUMMARY**

This program simulates the environment of a strapdown inertial measurement unit in an aircraft executing a user-specified series of idealised manoeuvres. The program generates a file containing a sequence of specific forces and angular velocities in body axes coordinates, or a file containing a sequence of integrated (in body axes) specific forces and angular velocities, or both. The sequences are time-tagged, and also include aircraft height. For "reference" purposes it generates a file containing a sequence of nominally true position, velocity, and attitude of the aircraft.



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## 1. INTRODUCTION

Simulations of Inertial Navigation Systems (INS) require a simulated environment to exercise the INS model. Specifically, the environment consists of the angular velocities and specific forces experienced by the INS, together with height. (Specific force is the nongravitational acceleration to which the body is subjected, and is measured by the accelerometers).

The flight profile generator FPG2 simulates the environment for a strapdown inertial measurement unit (IMU) in an aircraft executing a user-specified series of idealised manoeuvres. The simulated IMU environment is located at the centre of gravity of the aircraft, and the IMU axes are coincident with the aircraft body axes.

The program generates a file containing a sequence of specific forces and angular velocities in body axes coordinates. Optionally, it will generate a file containing a sequence of integrated (in body axes) specific forces and angular velocities, or both. This is useful for an IMU having integrating gyroscopes and accelerometers, when the FPG2 output is equivalent to "perfect" sensor output.

For "reference" purposes it generates a file containing a sequence of nominally true position, velocity, and attitude of the aircraft. The sequences are time tagged, and the IMU environmental data include aircraft height. FPG2 also generates an initialisation file for use by another program containing a simulation model SINS1 (reference 1) of a strapdown inertial navigation system which is to experience the FPG2 simulated environment.

A "flight" consists of a string of segments, during each of which one of the following manoeuvres may be carried out. A "Banked turn" is a change of heading achieved in the conventional way by banking the aircraft: it is at constant speed, and with constant elevation angle - it may be a climbing or a diving turn. A "Vertical turn" is a change in elevation angle at constant heading, and with wings level. "Straight flight" is movement along a constant heading flight path: elevation angle is constant, but the aircraft may be climbing or diving. A "Taxi turn" is a change of heading achieved by a rotation about the vertical axis through the aircraft: elevation angle is not necessarily zero, but is constant. Vertical turns, straight flight, and taxi turns may be at a constant (or zero) speed or with constant linear acceleration along the flight path.

Manoeuvres are idealised, in that total aircraft velocity relative to earth is along the aircraft's fore-and-aft axis. Banked and vertical turns are coordinated: sideslips and variation of angle of attack are not allowed. The purpose of this is to allow flight control to be of a simple open loop nature, and to exclude the need for simulation of vehicle aerodynamics and control system.

Manoeuvre specifications for the flight are prepared by the user as a data file. The program has an interactive facility for generating and editing its input data files. This facility can step FPG2 through a flight, one manoeuvre at a time: it allows specifications of individual manoeuvres to be altered during a flight, and the individual manoeuvres can then be repeated without restarting the whole flight.

Earth is modelled as a rotating ellipsoid, the parameters of which may be specified separately. The initial state of the aircraft, and the timing and output requirements for a simulation run, are specified by the user in the input file, along with the manoeuvre specifications.

An earlier version of this program was written in 1980 by R.B. Miller at the Aeronautical Research Laboratory (ARL), Melbourne, for the DEC-10 computer then in use. That version was never documented. In 1985/6, the author was attached to the Royal Aircraft Establishment, Farnborough. During this period, the program was converted to run on the Prime computer in use there, and the opportunity was also taken to make several improvements to the program's structure and facilities. These included the interactive program "WINDATA" which facilitates preparation of the data file for FPG2.

At ARL, FPG2 is now available on the Elxsi computer. WINDATA has been converted to operate as a subroutine of FPG2. Source code for the program is approximately 3800 lines of F77 Fortran.

## 2. NOTATION AND COORDINATE FRAMES

Notation used in this document is explained in Appendix 1. Several coordinate frames are employed: these and their relationships are illustrated in Appendix 2. The Navigation frame ( $N_1, N_2, N_3$ ) may be either the North, East, Down, Geographic frame ( $G_1, G_2, G_3$ ), or the Wander frame ( $W_1, W_2, W_3$ ), as is used by many modern INS's to avoid singularities at the poles, according to the user's requirements for wander angle data. When Wander frame is specified, the program provides wander angle with the reference outputs. However, flight over the poles cannot be simulated,

because the program uses the wander angle internally.

Aircraft Body (B1,B2,B3) axes are the fore-and-aft axis (forward +ve), the lateral axis (starboard +ve), and the normal axis respectively. These are often referred to as the roll, pitch, and yaw axes: the aircraft "rolls" about B1, "pitches" about B2, and "yaws" about B3.

Conventionally, rotations from Navigation to Body axes are defined as follows. "Heading" is the angle between North (G1) and the horizontal projection of B1. "Elevation angle" is the smaller angle between B1 and the horizontal. It is positive with nose up. "Bank angle" is the smaller angle through which the aircraft has rolled (positive clockwise) about B1, to take B2 from horizontal to its actual position. For present purposes, "Relative Heading" is here defined as the angle between W1 and the horizontal projection of B1: heading and relative heading are positive clockwise about Down.

The terms roll angle and pitch angle will at times be used here. They are to be interpreted as angles rotated about roll and pitch axes respectively. Because of the definitions of the manoeuvres used here, it happens that roll angle is equivalent to bank angle and pitch angle to elevation angle. It should be noted that this is not the general case. The same applies to roll and pitch rates (angular velocities about B1 and B2).

### 3. FLIGHT PROFILE GENERATION

A simulated flight is made up of segments, each containing one manoeuvre. Typically it may begin with a series of straight "flights" and taxi turns to correspond with the movements of the aircraft on the airfield, followed by acceleration along the runway, pitch up, climb out, accelerate, various banked turns to achieve required headings, and vertical turns for height changes, interspersed with periods of straight flight, and with along path acceleration to attain required speeds. This may be followed by an approach and landing sequence constructed in the same way.

Each manoeuvre segment is specified by its type and duration, and some of the following: heading or elevation angle change, linear acceleration along path, maximum allowed normal acceleration (the "g" of the turn), maximum allowed bank angle, maximum aircraft angular velocity about the particular axis, and the required angular acceleration of the aircraft about that axis.



The program is organised as a time-stepping simulation. The aircraft proceeds through its flight by executing sequences of rotations and along path linear accelerations, which the program calculates for each manoeuvre from the specifications of the manoeuvre. As the flight proceeds, the equations of the aircraft's motion are numerically integrated at the iteration rate specified by the user. Thus the aircraft position, velocity, and attitude are updated, and the IMU environment is calculated at the intervals required.

### 3.1 Calculation of Manoeuvre Parameters

At the beginning of each segment during a run, the program reads the manoeuvre specifications, together with any other internal information required, and calculates a series of roll (B1 axis) angular velocities, (for banked turn), or pitch (B2 axis) angular velocities (for vertical turn), or down (N3 axis) angular velocities (for taxi turn), as functions of time. For this purpose, each segment is divided into eight time sectors, during each of which the appropriate angular velocity is a linear function of time (or zero or constant). These data are used while running through the manoeuvre to calculate the Body to Navigation frame angular velocity.

In the turning manoeuvres, specification of time is optional: the program always allocates enough time to complete the manoeuvre as specified, and if the specified time is greater than that required, the balance of time is taken as straight flight.

#### 3.1.1 Banked turns

Banked turns are specified by: heading change required, maximum bank angle or B3 axis linear acceleration allowed, maximum roll angular velocity, roll angular acceleration, and total time.

For a coordinated banked turn:

$$\text{Rate of change of heading} = (g/V) \cdot \tan(\text{bank angle}),$$

where  $g$  is gravity and  $V$  is aircraft velocity. This relationship applies for any constant elevation angle - see Appendix 3.

A roll profile consisting of seven elements is calculated for the manoeuvre, consisting of a specified angular acceleration up to a required angular velocity,

which is held constant until a reverse angular acceleration is applied to reduce the angular velocity to zero, at which time the required roll angle has been achieved. This roll (bank) angle is held constant for a time, then the aircraft rolls out by a reversal of the roll-in process. It is assumed that the above bank/heading change relationship holds throughout, and timing of the changes in roll angular acceleration are calculated such that the required heading change is achieved.

In cases where the heading change or the angular acceleration are small, it may happen that the specified maximum angle or angular velocity cannot be attained without overshooting the heading. The program checks for this and reduces values if necessary. In extreme cases, such as a very small heading change with large bank angle specified, the program may decide to do a taxi turn instead. Similarly, if the forward velocity happens to be zero, the manoeuvre is undefined, and the program will select a taxi turn instead.

Either or both the maximum bank angle and the maximum B3 axis linear acceleration will be accepted as limits to the manoeuvre: the program selects the one which provides the less severity, subject to validity.

Calculation of roll profile is discussed in greater detail in Appendix 3.

### 3.1.2 Vertical turns

Vertical turns are specified by pitch angle change required, maximum allowed normal (B3) axis linear acceleration or pitch angular velocity, pitch angular acceleration, linear acceleration along path, and total time. B3 axis acceleration in this manoeuvre is that arising from turning effects only; gravity effects are additional.

A pitch profile consisting of three elements is calculated for the manoeuvre, consisting of a specified angular acceleration up to a required angular velocity, which is held constant until a reverse angular acceleration is applied to reduce the angular velocity to zero, at which time the required pitch angle change has been achieved.

For a coordinated pitching turn with zero bank:

$$\text{pitch rate} = YA/V$$

where  $Y_A$  is B3 axis linear acceleration caused by turning, and  $V$  is velocity along path. In calculation of the pitching profile, pitch rate is determined from the specified B3 acceleration using the above relationship, or from the specified rate, whichever is the lesser value. Velocity is taken as that at the start of the manoeuvre: if along-path acceleration increases velocity during the manoeuvre, B3 acceleration will exceed the specified maximum later in the turn. In the case where initial velocity is zero, the specified pitch rate is always used. These calculations are discussed in greater detail in appendix 4.

### 3.1.3 Straight flight

Straight flight is specified by a linear acceleration along the B1 axis (the path), and by total time: no calculation of parameters is required.

### 3.1.4 Taxi turns

Taxi turns are specified by heading change required, maximum allowed heading angular velocity, heading angular acceleration, linear acceleration along path, and total time.

A heading profile consisting of three elements is calculated for the manoeuvre, consisting of a specified angular acceleration up to a required angular velocity, which is held constant until a reverse angular acceleration is applied to reduce the angular velocity to zero, at which time the required heading angle change has been achieved.

Calculation of parameters is similar to that for a vertical turn, except that angular velocity is about the vertical axis. However, no limits are placed on lateral acceleration.

## 4. ANALYTICAL ASPECTS

Those quantities which are used in defining the state of the aircraft at any particular time are arranged in FPG2 as a state vector, which is propagated through the duration of the flight by numerical integration. The state vector contents are: velocity components, altitude, components of two quaternions (navigation to body axes, and navigation to earth axes), and components of integrated angular velocity and specific force.

Integration is performed by a fourth order or a second order Runge-Kutta routine. Fourth order is used when Body to Navigation frame angular velocity is non-zero, otherwise second order is used; this is to save computer loading.

#### 4.1 The State Vector and its Derivatives

Runge-Kutta routines require time derivatives of the elements of the state vector (SV). At each step, these are calculated from the current estimate of the appropriate SV elements, together with data calculated from the manoeuvre specifications.

Elements (1) to (3) of the SV are aircraft velocity (relative to Earth) components in Navigation axes. Derivatives are obtained from the equation (see appendix 5):

$$\{\dot{V}\}^N = C_B^N \{\dot{V}\}^B + \{\omega_{NB}\}^N \times \{V\}^N$$

where the elements of:

$\{V\}^N$  are in the SV;

$C_B^N$  are calculated from SV quaternion elements (6) to (9).

(See, for example, reference 1);

$\{\dot{V}\}^B$  are obtained from the manoeuvre specification;

$\{\omega_{NB}\}^N$  are calculated from the manoeuvre parameters.

See section 2.1 and appendix 6.

Element (4) of the SV is the along-path velocity of the aircraft. The derivative of this is the along-path acceleration, which is part of the manoeuvre specification.

Element (5) is altitude. The derivative of this is the vertical component of velocity, SV element (3).

Elements (6) to (9) are the components of the Navigation to Body frame quaternion. Derivatives of these are calculated from the quaternion differential

equation (see, for example, reference 2);

$$\dot{\bar{Q}}_{NB} = \frac{1}{2} [\underline{\omega}_{NB}]^N (*) \bar{Q}_{NB}$$

where  $[\underline{\omega}_{NB}]^N$  has been calculated as above.

Elements (10) to (13) are the components of the Navigation to Earth frame quaternion. Derivatives are calculated as above, except that  $[\underline{\omega}_{NE}]^N$  is used.  $[\underline{\omega}_{NE}]^N$  is calculated from aircraft velocity, altitude, latitude, and local Earth radii (see appendix 6). Earth radii are calculated as in reference 3 (see, also appendix 7).

Elements (14) to (16) are the components of integrated angular velocity of Body relative to Inertial frame, in body axes. Derivatives of these are the components of angular velocity, which are calculated by adding the angular velocities of the Body relative to Navigation frame, the Navigation relative to Earth frame, and the Earth relative to Inertial frame (see appendix 6).

Elements (17) to (19) are the components of integrated specific force in body axes. Derivatives of these are the components of specific force, which are calculated from the equation (see appendix 8):

$$[\underline{SF}]^B = [\dot{\underline{V}}]^B + C_N^B \{ [\underline{\omega}_{IB} + \underline{\omega}_{IE}]^N \times [\underline{V}]^N - [\underline{g}]^N \}$$

where all of the quantities except  $[\underline{g}]^N$  have been obtained as above.  $[\underline{g}]^N$  is calculated as in reference 4 (see also, appendix 7).

## 5. RUNNING THE PROGRAM

Input and output data files associated with FPG2 operation are listed and described below. The suffix .xxx is a string of three characters which are provided by the user to identify the particular run. All the input data files used must be in the same directory as the FPG2 executable file. The program requests the name of the (existing) directory where its output files will be written.

On starting FPG2, the program asks via the terminal :

Do you wish to run the flight profile generator (R/r)  
or to prepare the data file (P/p)?

The user responds with his requirement. Assuming the former is required, it then asks for the filename suffix (.xxx) for this run. Next, it asks the name of the output directory. Finally, it runs through the "flight", writing data to output files and the terminal according to the users requirements as specified in the input data file (INDATA.xxx). If interactive preparation of the input data file is required, the WINDATA routines are activated. (See below).

### 5.1 Input data files

Timing data, initial conditions, and manoeuvre specifications for a "flight" are to be in a file INDATA.xxx, which is essential unless the WINDATA routines are in use. An optional file of Earth data EZ.ERTHDATA may be provided if default values are not to be used, and if the "flight" is to start at other than the first manoeuvre segment, a late start file LSDATA.xxx may be provided.

#### 5.1.1 Input Data File INDATA.xxx

This file is essential for running the flight profile generator, and it is to be in the same directory as the FPG2 run file. The format of an INDATA file is shown below :

First line is timing data :

NOSEGS TICPERSEC STEPTICS SSPERIMU SSPERREF ... (continued)  
... SSPERDYN DOIMU DODYN ITTY IWAZ

Next line is initial conditions :

ALONGO ALATO ALFAO ALTO AROLO APICHO AHEAD0 AVTO

Following lines are manoeuvre data, one per manoeuvre, maximum 50 lines :

ITYP ITIM DHEAD DPITCH VTDOT YACC ROLMAX ANGVEL ANGACC

```

.      .      .      .      .      .      .      .      .
.      .      .      .      .      .      .      .      .
.      .      .      .      .      .      .      .      .
.      .      .      .      .      .      .      .      .
.      .      .      .      .      .      .      .      .

```

The variable names given above are those used in the program. Timing data are all "integer" quantities. Their meanings are :

NOSEGS            the total number of manoeuvres in the flight.  
TICPERSEC        a "tic" is the smallest time division used in the program : this is the number per second. Typically it may be set to 1000 or 1000000 - its magnitude does not affect the running of the program, only the timing resolution.  
STEPTICS        the number of "tics" per simulation step.  
SSPERIMU        the number of simulation steps between "IMU" data.  
SSPERREF        the number of simulation steps between "Reference" data.  
SSPERDYN        the number of simulation steps between "Dynamics" data.  
DOIMU            IMU output flag (0 for no output)  
DODYN            Dynamics output flag (0 for no output)  
ITTY            Terminal output flag (0 for no output, 1 for partial, 2 for full output)  
IWAZ            Wander azimuth flag (0 for Geographic axes, else w/azimuth).

Initial conditions are respectively Longitude, Latitude, Wander angle, Altitude, Bank angle (must=0), Elevation angle, Heading, Velocity along forward body axis. Angles are degrees, altitude metres (+ up), velocity metres/sec. All variables are "real" quantities.

Manoeuvre data are respectively : Type of manoeuvre (1 for banked turn, 2 for pitching turn, 3 for straight flight, 4 for taxi turn), Duration (seconds), Heading change, Pitch change, Along track acceleration, Maximum B3 axis linear acceleration, Maximum bank angle, Maximum angular velocity, Angular

acceleration. Units are degrees, metres, seconds. ITYP and ITIM are "integer" quantities, the rest are "real".

### 5.1.2 Interactive Preparation of an INDATA.xxx File

This facility uses a question and answer method with menus of options to provide a simple means of preparing the data file.

If this facility is selected when starting FPG2, the user is presented with the main menu of operations, which are to CReate, CHange, or SAve an INDATA file, or to QUIT the program :

```
Data filename: [path]/INDATA.[number] ( [...] optional )
CReate  [number] [path]           :Creates an INDATA file
CHange  [number] [path]           :Changes the specified file
SAve    [number] [path]           :Saves the current file
QUIT                                         :Quits the program
```

(User response is required here)

The response is the first two letters of the described operation, followed (except for QUIT) optionally by the up to three character string [number] to be inserted into xxx. This would usually be a number. If this is specified, it may optionally be followed by a directory pathname [path] into which the file may later be written. If the path is not specified, the current directory is used. The response and strings must be separated by one or more blank spaces.

The program holds the "current (INDATA) file" in its memory, under the "current filename". This is not permanently saved until the SAve command is used. The filenames INDATA.XXX and INDATA.999 are reserved and must not be assigned by the user. The name INDATA.XXX is used within the program to denote an "empty" file, to which no name has been assigned by the user, and into which no data has been read. INDATA.999 is used within the program for a file to which the user has not assigned a name, but which contains data.

Upon selection of the CReate operation, the program requests the user to specify in turn the various quantities required for the INDATA file, (see section 5.1.1) commencing with the timing and output specifications, then the initial



conditions, then the manoeuvres. This is a straightforward question and answer process, in which the program will ask the user all it needs to know.

The CHange operation allows the user to edit a file. If the user specifies a filename which already exists, that file will be read into memory as the current file. If a filename is not specified, the current filename and its contents will be used. If there is no specified or current filename, the user is referred to the CReate command. A file may be created with CHange, but this is not recommended. The CHange operation works like a line editor program — each manoeuvre is a line in the data file, and CHange has a pointer which marks the "current manoeuvre" line.

CHange has a menu of commands which allow the user to move the pointer to different manoeuvres, to display one or more manoeuvre specifications, to delete and to insert manoeuvres, and to change separately any of the components of a manoeuvre specification (a "segment" is a manoeuvre):

N	Next segment.
I	Insert after the current segment.
IB	Insert before the current segment.
D	Delete current segment.
DF	Delete all segments Forward of the current segment.
T	Top.
B	Bottom.
P	Print current segment.
PP	Print three segments. (one above to one below segment).
P (number)	Print (number) forward or back (-ve)
S (number)	go to Segment (number).
CD (number)	Change the duration to (number).
CH (number)	Change the Heading change to (number).
CP (number)	Change the Pitch change to (number).
CF (number)	Change Along-path Acceleration to (number).
CY (number)	Change the Yaw acceleration to (number).
CR (number)	Change Maximum Roll Angle to (number).
CW (number)	Change Maximum Angular Velocity to (number).
CA (number)	Change Angular Acceleration to (number).
RE	REturn
F	Initial values menu.
LO	LOok at current manoeuvre.
Z	Command list.

CH SUB-COMMAND : (User response is required here).

"Print" means display manoeuvre specifications on the terminal. User response is the command letter/s, followed if appropriate by the number.

The above menu of commands is presented to the user on entering CHange, and later as requested (Z). A sub-menu of commands is available (response F) to edit the initial values, i.e. the timing specifications and the initial conditions comprising the first two lines of the INDATA file :

Change commands & current initial values :

T	TICS per sec	1000000	S	Step length	(tics)	10000
A	Altitude (m)	100.0	L	Latitude	(deg)	-40.0
M	Longitude (deg)	140.0	N	Screen o/p	(2/1/0)	1
D	DYN output	(1/0) 0	E	DYN o/p	(steps)	0
I	IMU output	(1/0) 1	J	IMU o/p	(steps)	1
W	Wander azimuth	(1/0) 1	X	W/azim angle	(deg)	90.0
H	Heading (deg)	90.0	P	Pitch	(deg)	0.0
R	Roll (0)	0.0	V	Velocity	(m/sec)	0.0
O	Ref o/p (steps)	1000.				

Z Command List RE Return

F Sub-command: (User response is required here).

This menu shows the current values of the quantities, and a "typical" set is shown in the example. User response is the command letter, followed by a space, and the required new value.

One of the CHange commands (LO) allows the user to run the flight through the current manoeuvre, so the results of the manoeuvre or changes to its specification may be seen almost immediately. This is achieved by running the appropriate routines from the profile generator. An internal record is kept of the starting conditions for each segment, up to the last segment which has previously been run in that session, and has not subsequently had its specifications altered. When a run through the current manoeuvre is required, the run actually starts from the last unmodified manoeuvre: obviously, a change to a manoeuvre specification will alter the starting conditions for all subsequent manoeuvres.

The SAve operation is used to transfer the contents of the "current file" into permanent memory. If no filename is specified, the "current filename" is used,

provided it is legal. If no current filename has been specified (INDATA.999 in use), the user will be asked for a filename, or to have a default filename used. In the latter case, the value of xxx is the lowest available number not already used.

The QUIT operation is the means of exiting the program. Before doing so, it prompts the user to SAVE his file, if required. QUIT does not save any files.

### 5.1.3 Earth Data File EZ.ERTHDATA.

This is optional, but if required, should be in the same directory as the FPG2 run file. It contains values for: Equatorial radius (metres), the inverse of ellipticity, Earth rate (radians/sec), Equatorial sea-level gravity ( $\text{m/sec}^2$ ), Gravity latitude coefficient, gravity altitude coefficient, gravity altitude squared coefficient, and North gravity component coefficient. A typical example of the file content is:

```
6378135.0 298.26 0.0000729211515 9.780333 0.0052884 2.014 3.05 0.00523
```

These values are those from Appendix 7 which are also the default values.

### 5.1.4 Late Start Data File LSDATA.xxx.

This file need only be prepared if it is required to start the flight other than at the first manoeuvre. It contains the number of the first manoeuvre to be performed, the start time, and the state vector values at that time. The form of the file, which must be in the same directory as the FPG2 run file, is:

```
<start manoeuvre no.> <hours> <minutes> <seconds>
<state vector elements (1) to (5)>
<state vector elements (6) to (9)>
<state vector elements (10) to (13)>
```

The quantities on the first line are "integer", the others are double precision "real".

At the end of each manoeuvre, one of the output routines records these quantities in compatible format in an output file SEGOUT.xxx (see below).

## **5.2 Output data files**

Some of the following files will be created and written in the output directory as specified by the user. Existing files of the same name will first be deleted.

### **5.2.1 Reference output REFOUT.xxx**

This ASCII file consists of a single header line followed by lines of "reference" or "truth" flight profile data. The data consists of: Time; Manoeuvre no.; Longitude; Latitude; Wander angle; Altitude (sign set to positive up); Velocity components north, east, down; Navigation to body frame bank, elevation, and heading angles. Units are seconds, degrees, and metres.

### **5.2.2 Integrated sensor environment IMUOUT.xxx**

Production of this file is optional. Each write to this file consists of an array of eight binary double precision quantities, being respectively the three components of integrated body axes angular velocity, three components of integrated body axes specific force, the altitude, and a time tag. The first six quantities correspond to "perfect" outputs from integrating rate gyroscopes and integrating accelerometers respectively.

### **5.2.3 Non-Integrated sensor environment DYNOUT.xxx**

Production of this file is optional. The contents are similar to IMUOUT.xxx, except that the instantaneous values of the (body axes) components of angular velocity and specific force are written, along with altitude and time. The first six quantities correspond to "perfect" outputs from rate gyroscopes and accelerometers respectively.

### **5.2.4 Information file SEGOUT.xxx**

The first part of this file contains a record of initial conditions and manoeuvre data, in a more readable form than in the input data files. At the start of each manoeuvre, the times at which the various body rate changes will occur are recorded, and in the case of a banked turn, the heading changes that will occur in each sector of the manoeuvre.

At the start of each manoeuvre segment, the segment number, time, and state vector components are recorded in a form suitable to be copied into a late start data file.

#### **5.2.5 FPSIN2 Initialisation file SIMDATA.xxx**

FPSIN2 is a handler program for a simulation (SINS1 - reference 1) of a strapdown INS. It reads IMUOUT.xxx to obtain sensor environment data, and reads REFOUT.xxx to obtain its truth data. SIMDATA.xxx is only written if IMUOUT.xxx is written, and contains information required by FPSIN2 to initialise itself, namely the initial conditions of position, velocity, and attitude, the simulation step time, the total run time, and the time interval between reference data.

### **6. PROGRAM DESCRIPTION**

The main program module is FPGEXEC, shown in figure 1, which is entered when FPG2 is started. On entry, this asks via the terminal if the user wishes run the profile generator, or to prepare the data file. If the former, it calls FPGINIT, the initialisation routine, then steps through the flight with repeated calls to FPGLOOP until the flight is completed. Finally it tidies up with a call to FPGOUTS, the output controller routine. If the user wishes to prepare a data file, FPGEXEC calls the WINDATA routines (see section 6.4, below).

#### **6.1 Initialisation**

This is controlled by FPGINIT, shown in figure 2. This routine reads the input data files, sets up initial values of the state vector and other variables, counters and indices used in the program, and initiates preparation of output files by a call to FPGOUTS, the output controller.

If there is an earth data file, the values in it are used, otherwise a set of default values is used. Files suffix xxx is requested at the terminal. The input data file INDATA.xxx is opened and initial conditions are read. Existence of a late start data file is checked: if present, the data to allow the flight to begin at other than the first manoeuvre and time zero, are read.

Manoeuvre specification data are then read from INDATA.xxx, and a call is made to FPGOUTS to record the input data in a readable form in an output file.

The initial state vector and variables are then set up, and FPGOUTS is called to prepare the output files.

## 6.2 Flight Profile Generation

Routine FPGLOOP, shown in figure 3, is effectively a part of the program executive: it is entered at each step of the simulated flight.

If the entry is at the start of a new manoeuvre, appropriate initialisation for the manoeuvre is performed, including a call to the relevant manoeuvre setting up routine: HTSET for a banked turn, VTSET for a pitching turn, SFSET for straight flight, or TTSET for a taxi turn. It then calls FPGOUTS for recording of the manoeuvre characteristics and the state vector. Functions of these routines are described in Section 3.1.

The simulation step is then performed by a call to FPGSTEP, after which the timing is updated, and a call is made to FPGOUTS for routine output as required.

Routine FPGSTEP, shown in figure 4, is also a serial part of the program executive. Its function is to call the integration routine and to test for completion of separate manoeuvres and of the whole flight. In a simulation step which includes one or more time sector (section 3.1) boundaries, extra calls to the integration routine are made, so that separate integrations are made between the beginning of the step, the time sector boundaries, and the end of the step, except in cases where those times are zero or a very small (SMALLTIME in FPGINIT) fraction of the total step time. If the manoeuvre or the flight will be completed after the present step, flags are set for action by FPGLOOP or FPGEXEC respectively.

FPGSTEP calls a simple integration selection routine FPGINTG, whose function is only to select whether the current integration will be performed by second order or fourth order integration. If the vehicle is in straight flight, or the straight flight part of another manoeuvre type, second order is selected, otherwise fourth order. It is possible to force fourth order always, by having a negative value in either DOIMU or DODYN (see section 5.1.1).

FPGINTG, shown in figure 5, calls either RUKU4 or RUKU2, for fourth or second order Runge-Kutta integration. These routines call COMVAR, which is the calculation part of the program: it calculates all the variables required to do the simulation steps, as described in section 4. They also call a routine FF which returns the derivatives of the state vector elements as required by the integration process.

### 6.3 Output

All output from FPG2 other than terminal messages is performed under the control of FPGOUTS, which is shown in figure 6. This asks via the terminal for the (existing) directory name where the output files are to be written. It creates and writes to several files as follows:

REFOUT.xxx	an ASCII file of "reference" or "true" flight profile data.
IMUOUT.xxx	(optional) binary file of integrating sensor data.
DYNOUT.xxx	(optional) binary file of non-integrating sensor data.
SEGOUT.xxx	an ASCII file recording input, and state vector data.
SIMDATA.xxx	as ASCII file of initialisation data for FPSIN2.

On the first call from FPGINIT, the output files except SIMDATA.xxx are opened, routine HEADER is called to write headers to REFOUT.xxx and SEGOUT.xxx then routine CHECK is called to write the input data in SEGOUT.xxx.

On the second call from FPGINIT, routine OUTREF is called to write the initial reference output in REFOUT.xxx.

At the beginning of each manoeuvre, on a call from FPGLOOP, routine OUTSEG is called to write the state vector into SEGOUT.xxx.

At the end of every step through the flight, there is a call from FPGLOOP upon which FPGOUTS determines whether output is required for each of the files REFOUT.xxx, IMUOUT.xxx, and DYNOUT.xxx. The outputs are prepared and written by calls to routines OUTREF, OUTIMU, and OUTDYN respectively, as required.

After the final step of the simulation, there is a call from FPGEXEC, upon which the file SIMDATA.xxx is opened, and written to by a call to OUTSIM. All open files are then closed.

### 6.4 Preparation of Data File

Iterative preparation of the data file is under the control of routine WINDATA, shown in figure 7. This displays the main menu, then calls either CREATE, CHANGE, or SAVE, according to the user's response to the menu.

CREATE, shown in figure 8, first calls a routine DECODE, which interprets

the pathname and suffix (if any) specified by the user. It asks the user what values are required for timing, output, and initial conditions, then it makes calls to LINE, when each call asks the user for the specifications of a single manoeuvre, until the "flight" data is complete. The flight specifications are stored in memory in a format suitable for writing to the INDATA file. CREATE cannot edit data.

Editing of INDATA file contents is done by CHANGE, shown in figure 9. This calls DECODE as above, and if there is an existing INDATA file, its contents are read and stored by routine READ. According to the user's responses to CHANGE's main menu, the relevant manoeuvre data values in the stored INDATA image are altered, or manoeuvres can be added or deleted. Manoeuvres are added by calls to LINE. After any change in a manoeuvre's data, all the data for that manoeuvre is written to the terminal by routine WRLIN. Changes to timing or initial conditions are done by calling routine START, which puts up its own menu for such changes. If the user wishes to examine the effect, for example, of a change in a manoeuvre specification, CHANGE will call routine LOOK, which in turn calls FPGINIT and FPGLOOP as necessary to run through the current manoeuvre.

Routine SAVE calls DECODE as above, and performs error checking on the filename and pathname to ensure they are usable. If so, it opens, writes, and closes the INDATA file.

## 7. SCOPE FOR FURTHER DEVELOPMENT

In its present form this program generates a rather idealised environment in that the aircraft's banked and vertical turns are coordinated, and there are no sideslips or angle of attack variations. This is adequate for investigation of many of the effects of INS sensor errors and characteristics, and system misalignments. According to whatever application the program may be put, various enhancements could be incorporated without losing the utility of the open loop flight control.

Gust and vibration effects could be introduced. This could be done in COMVAR at the stage where the Body to Navigation angular velocities are calculated (for angular effects), and where the Body accelerations are calculated (for linear effects). These effects would preferably be generated at the acceleration level from zero mean sequences of appropriate variance, filtered to give required spectral characteristics.



Rotational freedom of the IMU relative to the Body could be introduced. At the simplest level, this could amount to a constant transformation of environment components from Body to IMU axes. A predetermined (with respect to time) relationship could be introduced in COMVAR as above. At a higher level, another quaternion set representing the Body to IMU rotation could be added to the state vector, and updated using the relative angular velocity: this would allow simulation of finite rotational stiffness between Body and IMU.

The environments of one or more extra IMU's at locations remote from the body centre of gravity could be simulated by incorporating the appropriate lever arm effects into the calculation of angular velocities, at each location. This would require additional elements in the state vector for each IMU axis, and the addition of output facilities for each IMU, similar to the present IMUOUT or DYNOUT and their associated routines.

Addition of further manoeuvre types may be found useful. As an example the addition of the taxi turn required only the inclusion of the set-up routine TTSET, an extra conditional branch in FPGLOOP and in COMVAR, and a minor addition to the CHECK routine. An addition to WINDATA was also made to include the manoeuvre in its repertoire.

Closure of the position and/or attitude control loop may be found useful if, for example, it were required that the vehicle fly over particular waypoints. This would require the incorporation of a segment to estimate changes to the angular velocity and/or linear acceleration during the flight, and to incorporate them at the appropriate points in COMVAR.

## 8. ACKNOWLEDGMENT

Thanks are due to R. Hankey, for his assistance in converting the program to run on the RA-7 Prime computer, and who wrote most of the "WINDATA", the interactive part of the program used for data file preparation.

# REFERENCES

No.	Author	Title
1	R.B. Miller	SINS1: A model of a strapdown inertial navigation System. ARL-SYS-TM-101 Jul 1989
2.	R.B. Miller	Strapdown INS: an Algorithm for Attitude and Navigation Computations. ARL-SYS-REP-23 October 1980.
3.	M. Kayton & W.R. Fried	Avionics Navigation Systems. J. Wiley & Sons 1964.
4.	W.S. Widnall & P.A. Grundy	Inertial Navigation System Error Models. Intermetrics. Inc.: TR-03-73, May 1973.

## APPENDIX 1. NOTATION

$\underline{U}$	a vector
$[\underline{U}]^X$	vector $\underline{U}$ in X frame coordinates
$\left[\frac{d\underline{U}}{dt}\right]_X$	rate of change of $\underline{U}$ with time with respect to frame X
$\left[\frac{d\underline{U}}{dt}\right]_X^Y$	is $\left[\frac{d\underline{U}}{dt}\right]_X$ in Y frame coordinates. N.B. $\left[\frac{d\underline{U}}{dt}\right]_X^X = [\dot{\underline{U}}]^X$
$\bar{Q}_{XY}$	quaternion representing rotation from X to Y frame
$C_X^Y$	direction cosine matrix: X to Y coordinates
$\underline{\omega}_{XY}$	angular velocity of Y frame relative to X frame
$\underline{V}_{XY}$	velocity of Y frame relative to X frame
$\underline{g}$	gravity vector
$\omega_E$	scalar Earth rotation rate about polar axis

## APPENDIX 2. COORDINATE FRAMES

Axes sets are defined as follows:

**INERTIAL (I)** - having I3 along Earth's axis of rotation. There is no need to define I1 and I2.

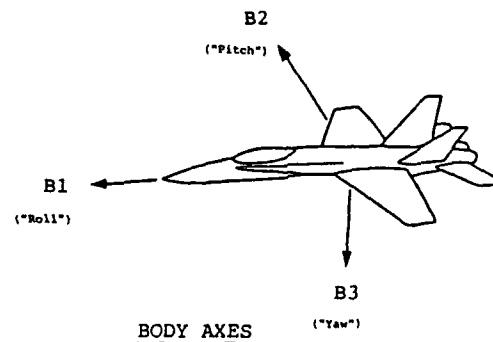
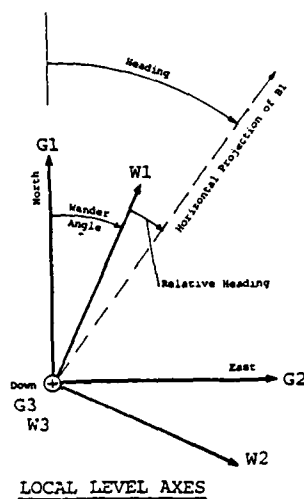
**EARTH (E)** - has E3 along Earth axis of rotation, E1 and E2 are in the equatorial plane, with E1 at 0 degrees longitude.

**GEOGRAPHIC (G)** - having G1 and G2 as local level North and East, and G3 down. The origin is at the vehicle centre of gravity.

**WANDER AZIMUTH (W)** - is obtained by a positive rotation of the G set about Down, through the Wander Angle. The origin is at the vehicle centre of gravity.

**NAVIGATION (N)** - may be the G frame or the W frame, according to user requirements. The relationship between G and W frames is illustrated below.

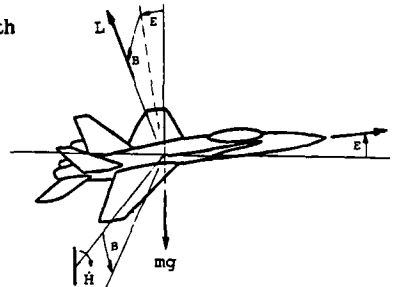
**BODY (B)** - is the set of aircraft axes, having B1 fore-and-aft (forward positive), B2 the lateral axis (starboard +ve), and B3 the normal axis, as illustrated below. These are often referred to as the roll, pitch, and yaw axes respectively: the aircraft "rolls" about B1, "pitches" about B2, and "yaws" about B3.



### APPENDIX 3. CALCULATION OF BANKED TURN PARAMETERS

#### A3.1 Heading Rate and Normal (B3) Linear Acceleration

Consider an aircraft of mass  $m$ , with bank angle  $B$ , constant elevation angle  $E$ , velocity along path (i.e. along B1 axis)  $V$ , with lift  $L$  perpendicular to the path, and heading rate  $\dot{H}$ . All drag is along the path.



Lift may be regarded as having components  $L \sin(B)$  along the radius of the turn, and  $L \cos(B)$  perpendicular to the radius of the turn and the path.

The horizontal projection of the flight path is circular, with speed  $V \cos(E)$ , therefore:

$$L \sin(B) = m \dot{H} V \cos(E)$$

also, by resolution of forces perpendicular to the flightpath and turn radius:

$$L \cos(B) = mg \cos(E)$$

from these we get heading rate:  $\dot{H} = \frac{g \tan(B)}{V}$

and linear acceleration along B3 is  $Y_A = L/m = \frac{g \cos(E)}{\cos(B)}$

#### A3.2 Heading Changes

These are obtained by integration of the heading rate equation above. In the rest of this appendix, roll angular velocity and roll angular acceleration are discussed. For conformity with this, the term roll angle will be used instead of bank angle.

For constant roll angle  $R$ , over time  $T$ , heading change is

$$H = \frac{g T \tan(R)}{V}$$

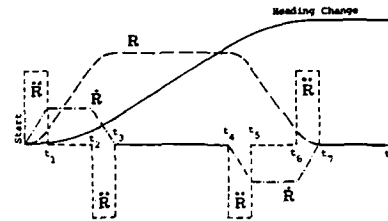
For constant roll angular velocity  $\dot{R}$ , from angle  $r_1$  to  $r_2$ :

$$H = \frac{g}{V \dot{R}} \log_n \left[ \frac{\cos(r_1)}{\cos(r_2)} \right]$$

For constant roll angular acceleration, the equation is integrated numerically, by a utility INTANN.

### A3.3 Roll Profile

The diagram shows a profile of roll angle ( $R$ ), angular velocity ( $\dot{R}$ ), and angular acceleration ( $\ddot{R}$ ): in subroutine HTSET, the program calculates the times at which the changes in  $\ddot{R}$  occur, in order that the aircraft undergoes the required change of heading during the turn.



HTSET first decides whether the maximum roll angle is limited by the specified maximum B3 axis linear acceleration or by the specified angle. This is done by comparing YA calculated as above with the specified acceleration. The roll angle  $R$  corresponding with the lower value is used.

HTSET then checks that  $\dot{R}$  is achieved before the roll angle is half of  $R$ . In the diagram, time taken to achieve  $\dot{R}$  is  $t_1$ , where:

$$t_1 = \dot{R} / \ddot{R}$$

however, the time  $t$  to reach half of  $R$  is given by  $(\frac{1}{2}R) = \frac{1}{2} \ddot{R} \cdot t^2$ , i.e.:

$$t = (R / \ddot{R})^{1/2}$$

If  $t$  is greater than  $t_1$ , HTSET puts  $t_1 = t_2 = t$ , otherwise:

$$t_2 = t_1 + (\text{total roll} - \text{roll from } t_0 \text{ to } t_1 - \text{roll from } t_2 \text{ to } t_3) / \dot{R}$$

$$\text{i.e.:} \quad t_2 = \dot{R} / \ddot{R} + (R - 2 \cdot \frac{1}{2} \ddot{R}^2 / \ddot{R}) / \dot{R}$$

$$\text{i.e.:} \quad t_2 = R / \dot{R}$$

$$\text{Then, in either case,} \quad t_3 = t_1 + t_2$$

HTSET then checks that this roll angle can be established before half of the heading change has been achieved: if it is assumed that  $\dot{R}$  is constant from zero to the maximum value  $R$ , heading change is given by the equation in A3.2:

$$H = \frac{R}{\dot{R}} \log_n |1 / \cos(R)|$$

if this is not less than half the required heading change, R is reduced by steps, repeating the checks above, until the heading change is less than half.

Having established a maximum roll angle consistent with the manoeuvre specifications, the remaining times are established. The exact value of heading change achieved during the roll-in (subject to the accuracy of the INTANN utility) is calculated. The heading change during roll-out is the same. The heading change at maximum roll angle is the total heading changes less these amounts. Heading rate at maximum roll is calculated as in A3.1, and the time follows.

The times when acceleration changes occur are each converted to an integer number of simulation step periods plus the remaining fractional part of a step period.

While running through the manoeuvre, the program obtains roll rates from equations of the form:

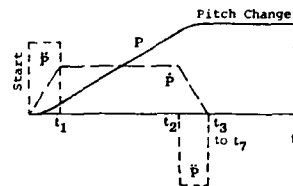
$$\dot{R} = X(k) + Y(k) \cdot t$$

where t is time into the manoeuvre and k is an index applicable to one of the eight manoeuvre elements.

HTSET prepares the values of X and Y, allowing for the direction of R: for example, in element (1), from t to t,  $X(1) = 0$  and  $Y(1) = \dot{R}$ .

#### APPENDIX 4. CALCULATION OF VERTICAL TURN PARAMETERS

The diagram shows a profile of Pitch angle ( $P$ ), angular velocity ( $\dot{P}$ ), and angular acceleration ( $\ddot{P}$ ): in subroutine VTSET, the program calculates the times at which the changes in  $\dot{P}$  occur, in order that the aircraft undergoes the required change of pitch during the turn.



VTSET first decides whether the maximum pitch rate is limited by the specified maximum normal acceleration or by the specified rate. This is done by comparing the pitch rate calculated from velocity and specified normal acceleration with the specified pitch rate. The lower value is used. Normal acceleration in this manoeuvre is that arising from turning effects only; gravity effects are additional.

VTSET then checks that the maximum pitch rate is achieved before half the required pitch angle change. This procedure is similar to that used in HTSET for bank angle changes:

if maximum rate is achieved, then:

$$t_1 = \dot{P} / \ddot{P}, \text{ and } t_2 = P / \dot{P};$$

otherwise:

$$t_1 = (P / \ddot{P})^{1/2} = t_2;$$

and, in either case:

$$t_3 = t_1 + t_2;$$

then  $t_4$  to  $t_7$  are set equal to  $t_3$ .

As in HTSET, times when acceleration changes occur are each converted to an integer number of simulation step periods plus the remaining fractional part of a step period. Also similarly, VTSET prepares values of  $X$  and  $Y$  for equations of the form:

$$\dot{P} = X(k) + Y(k) \cdot t$$



# APPENDIX 5. VELOCITY DERIVATIVE

Apply Coriolis' equation to velocity of aircraft relative to Earth (V):

$$\left[ \frac{d\underline{V}}{dt} \right]_N = \left[ \frac{d\underline{V}}{dt} \right]_B + \underline{\omega}_{NB} \times \underline{V}$$

in Navigation frame coordinates:

$$\left[ \frac{d\underline{V}}{dt} \right]_N^N = C_B^N \left[ \frac{d\underline{V}}{dt} \right]_B^B + [\underline{\omega}_{NB}]^N \times [\underline{V}]^N$$

i.e.:

$$[\dot{\underline{V}}]^N = C_B^N [\dot{\underline{V}}]^B + [\underline{\omega}_{NB}]^N \times [\underline{V}]^N$$

in the program,

$$[\underline{V}]^N = \begin{bmatrix} \text{STATE (1)} \\ \text{STATE (2)} \\ \text{STATE (3)} \end{bmatrix}$$

and

$$[\dot{\underline{V}}]^B = \begin{bmatrix} \text{VTDOT} \\ 0 \\ 0 \end{bmatrix}$$

VTDOT is the along path acceleration, which is specified as a manoeuvre parameter.

## APPENDIX 6. FRAME RATES

6.1 Angular Velocity of Body Relative to Navigation Frame  $[\omega_{NB}]^N$ 

Consider the rotations Heading H (or Relative Heading H'), Elevation (E), Bank (B) in that order, from Navigation to Body axes, and their rates  $\dot{H}$ ,  $\dot{E}$ ,  $\dot{B}$ , as shown in the diagram.

Inspection of the diagram shows that:

$$[\omega_{NB}]^N = \begin{bmatrix} \cos(E)\cos(H) & -\sin(H) & 0 \\ \cos(E)\sin(H) & \cos(H) & 0 \\ -\sin(E) & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{B} \\ \dot{E} \\ \dot{H} \end{bmatrix}$$

By definition:

$$\text{heading} = \text{rel. heading} + \text{wander angle}$$

$$\text{i.e.} \quad H = H' + A$$

$$\text{therefore} \quad \dot{H} = \dot{H}' + \dot{A}$$

and for constant heading,  $\dot{H}' = -\dot{A}$ , which is either 0 or  $-v_E \frac{\tan(LAT)}{(R_p+h)}$  as below.

A6.2 Angular Velocity of Navigation Relative to Earth Frame  $[\omega_{EN}]^N$ 

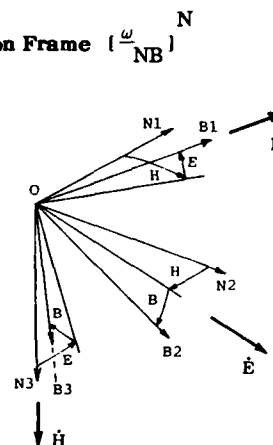
This is discussed in, for example, reference 1; the result is:

$$[\omega_{EN}]^N = \begin{bmatrix} v_E \frac{\cos(A)}{(R_p+h)} - v_N \frac{\sin(A)}{(R_m+h)} \\ -v_N \frac{\cos(A)}{(R_m+h)} - v_E \frac{\sin(A)}{(R_p+h)} \\ \dot{A} - v_E \frac{\tan(LAT)}{(R_p+h)} \end{bmatrix}$$

where  $v_N$  and  $v_E$  are North and East components of velocity,  $h$  is altitude, and  $R_m$ ,  $R_p$ , are as in Appendix 7.

If  $N$  is the Geographic (NED) frame, then  $\dot{A} = A = 0$ ,

if  $N$  is the Wander azimuth frame, then  $\dot{A} = v_E \frac{\tan(LAT)}{(R_p+h)}$



### A6.3 Angular Velocity of Earth Relative to Inertial Frame $(\underline{\omega}_{IE})^N$ .

This is also discussed in, for example, reference 1: the result is:

$$(\underline{\omega}_{IE})^N = \begin{bmatrix} \cos(\text{LAT}) \cos(A) \\ -\cos(\text{LAT}) \sin(A) \\ -\sin(\text{LAT}) \end{bmatrix} \cdot \Omega_E$$

where  $A = 0$  if  $N$  is the Geographic frame.

### A6.4 Angular Velocity of Body Relative to Inertial Frame $(\underline{\omega}_{IB})^B$ .

In Navigation coordinates :  $(\underline{\omega}_{IB})^N = (\underline{\omega}_{IE})^N + (\underline{\omega}_{EN})^N + (\underline{\omega}_{NB})^N$

and in Body coordinates :  $(\underline{\omega}_{IB})^B = C_N^B (\underline{\omega}_{IB})^N$

## APPENDIX 7. EARTH CHARACTERISTICS

### A7.1 Local Earth Radii

Reference 3 gives formulae for Local Earth Radii:

$$\text{Meridional radius: } R_m = \frac{R_E (1 - e^2)}{(1 - e^2 \sin^2(\text{LAT}))^{3/2}}$$

$$\text{Prime radius: } R_p = \frac{R_E}{(1 - e^2 \sin^2(\text{LAT}))^{1/2}}$$

where  $R_E$  is equatorial radius,  
 $e^2$  is (eccentricity)<sup>2</sup>.

Ellipticity or flattening is  $e$ , and  $e^2 = e(2 - e)$ .

WGS-72 has  $R_E = 6378135$  metres, and  $e = 1/298.26$

### A7.2 Gravity

Reference 4 quotes the following gravity formulae :

$$\text{North component : } g_n = g_0 (0.00000082.k.\sin(2.\text{LAT}))$$

$$\begin{aligned} \text{Down component : } g_d = g_0 (1 + 0.0052884.\sin^2(\text{LAT}) - 0.0000059.\sin^2(2.\text{LAT}) \\ - 0.0003157.k + 0.00000045.\sin^2(\text{LAT}).k + 0.000000075.k^2) \end{aligned}$$

where  $g_0$  is equatorial gravity,  $k$  is altitude in km., and LAT is latitude.  
 A value of 9.78049 metres/sec<sup>2</sup> is given for  $g_0$ .

Putting  $H = h/R_E$ , with  $h$  in metres and  $R_E$  as above, these formulae may be rewritten:

$$g_n = g_0 \cdot 5.23 \times 10^{-3} \cdot H \sin(2.\text{LAT})$$

$$\begin{aligned} g_d = g_0 (1 + 5.2884 \times 10^{-3} \sin^2(\text{LAT}) - 2.0136.H + 3.05.H^2 \\ - 5.9 \times 10^{-6} \sin^2(2.\text{LAT}) + 2.87 \times 10^{-3} H \sin^2(\text{LAT})) \end{aligned}$$

At an altitude of about 12800 metres or 42000 feet, the value of  $H$  is approximately 0.002.

For altitudes between zero and about 42000 feet, and any latitude, the total of the last two terms in the  $g_d$  formulae is always less than about  $6 \times 10^{-6}$  and in this program they are neglected.

### APPENDIX 8. SPECIFIC FORCE

Specific force is obtained from the navigation equation, the derivation is repeated here for completeness.

By definition,  $\underline{SF} = \left[ \frac{d^2 \underline{R}}{dt^2} \right]_I - \underline{g}_m$  ( $\underline{g}_m$  is mass attraction).

Apply Coriolis equation to velocity vector  $\underline{V}$ :

$$\left[ \frac{d\underline{V}}{dt} \right]_I = \left[ \frac{d\underline{V}}{dt} \right]_B + \underline{\omega}_{IB} \times \underline{V}$$

Apply Coriolis equation to position vector  $\underline{R}$ :

$$\left[ \frac{d\underline{R}}{dt} \right]_I = \left[ \frac{d\underline{R}}{dt} \right]_E + \underline{\omega}_{IE} \times \underline{R}$$

i.e.

$$\left[ \frac{d\underline{R}}{dt} \right]_I = \underline{V} + \underline{\omega}_{IE} \times \underline{R}$$

Differentiate relative to Inertial frame :

$$\left[ \frac{d^2 \underline{R}}{dt^2} \right]_I = \left[ \frac{d\underline{V}}{dt} \right]_I + \underline{\omega}_{IE} \times \left[ \frac{d\underline{R}}{dt} \right]_I$$

Substitution gives:

$$\left[ \frac{d^2 \underline{R}}{dt^2} \right]_I = \left[ \frac{d\underline{V}}{dt} \right]_B + \underline{\omega}_{IB} \times \underline{V} + \underline{\omega}_{IE} \times \underline{V} + \underline{\omega}_{IE} \times \underline{\omega}_{IE} \times \underline{R}$$

put  $\underline{g} = \underline{g}_m - \underline{\omega}_{IE} \times \underline{\omega}_{IE} \times \underline{R}$ , and substitute in specific force equation:

$$\underline{SF} = \left[ \frac{d\underline{V}}{dt} \right]_B + (\underline{\omega}_{IB} + \underline{\omega}_{IE}) \times \underline{V} - \underline{g}$$

Expressed in Body coordinates:

$$[\underline{SF}]^B = \left[ \frac{d\underline{V}}{dt} \right]^B + C_N^B \{ [\underline{\omega}_{IB} + \underline{\omega}_{IE}]^N \times [\underline{V}]^N - [\underline{g}]^N \}$$

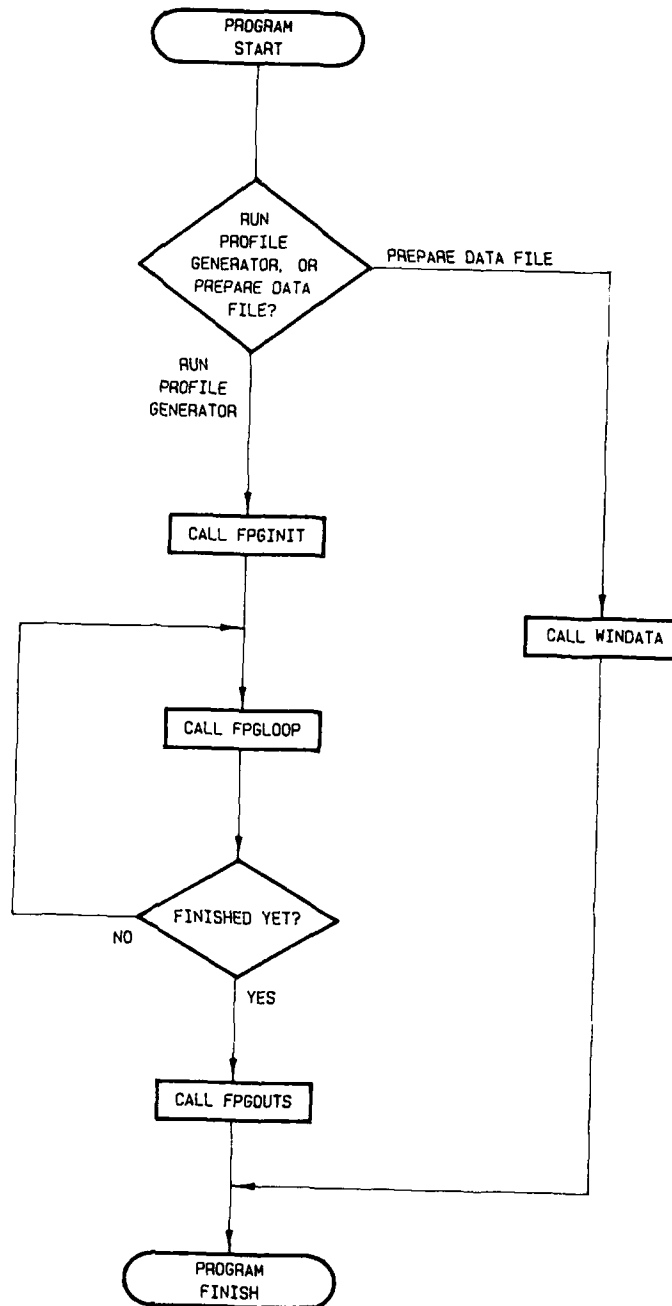


FIG. 1. FPGEXEC

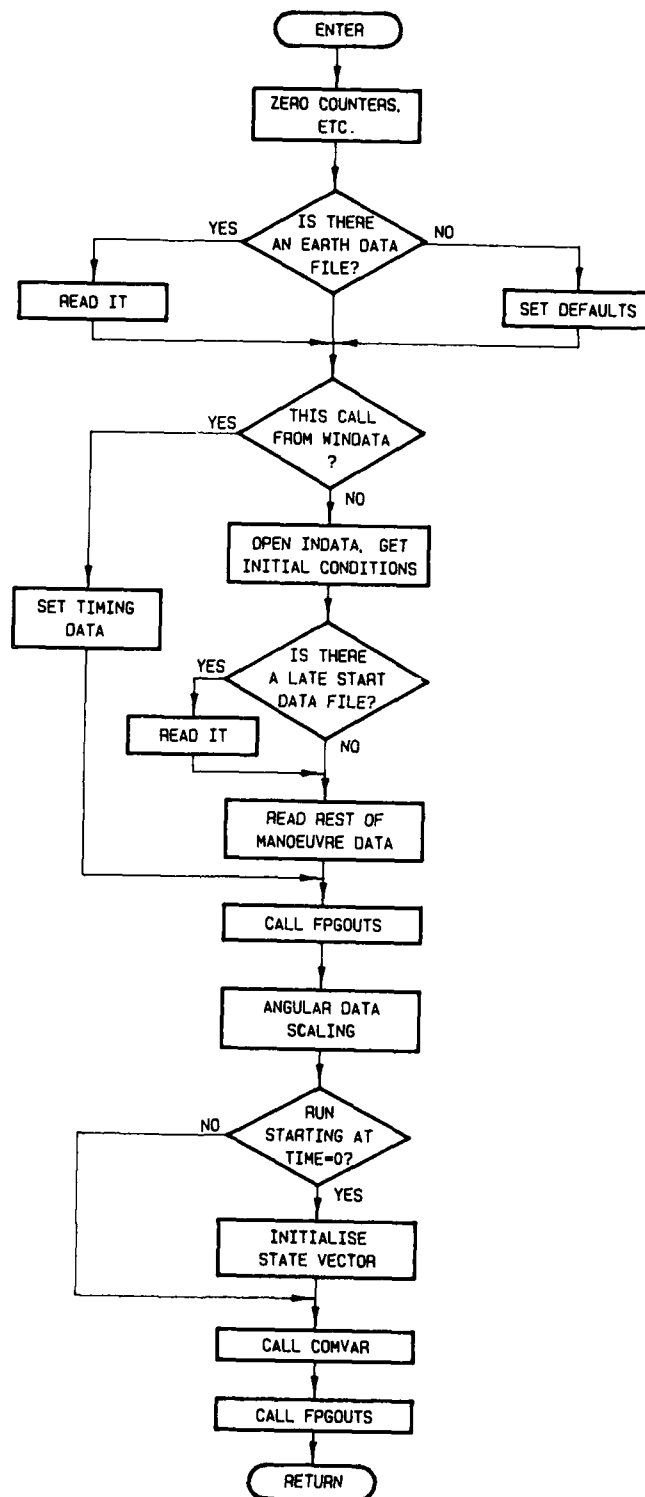


FIG. 2. FPGINIT

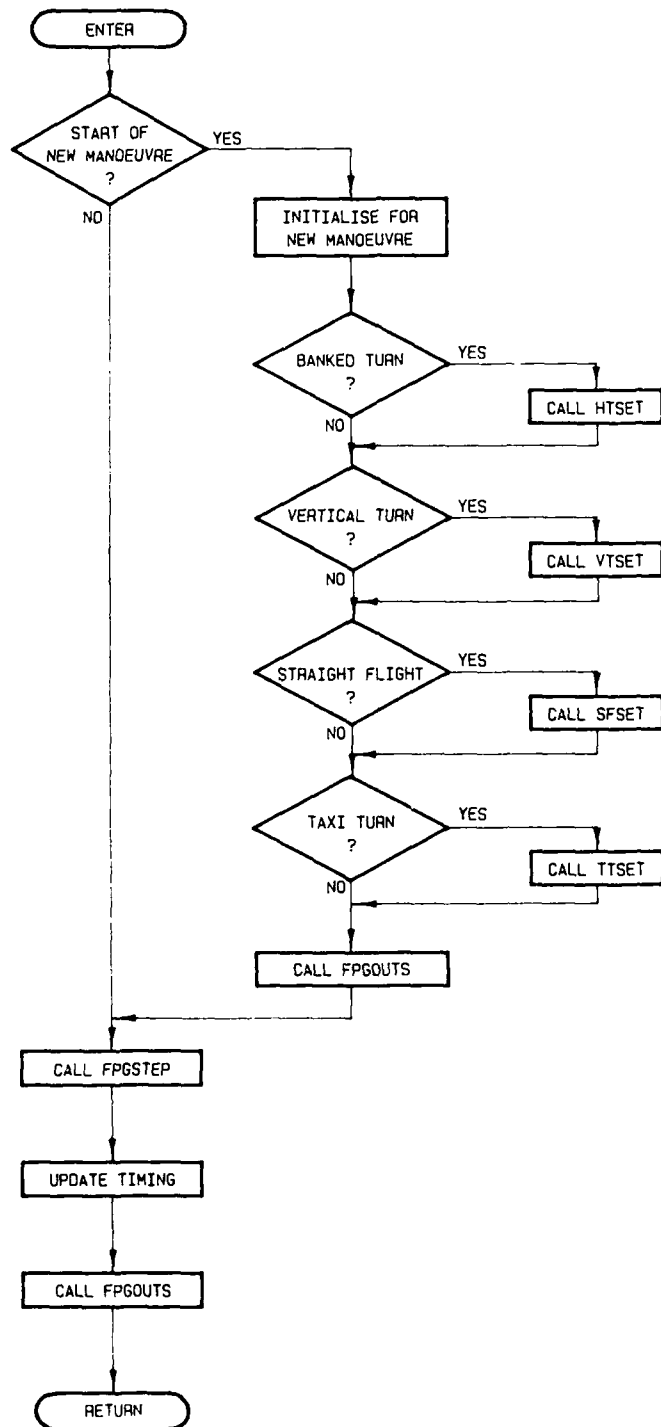


FIG. 3. FPGLOOP



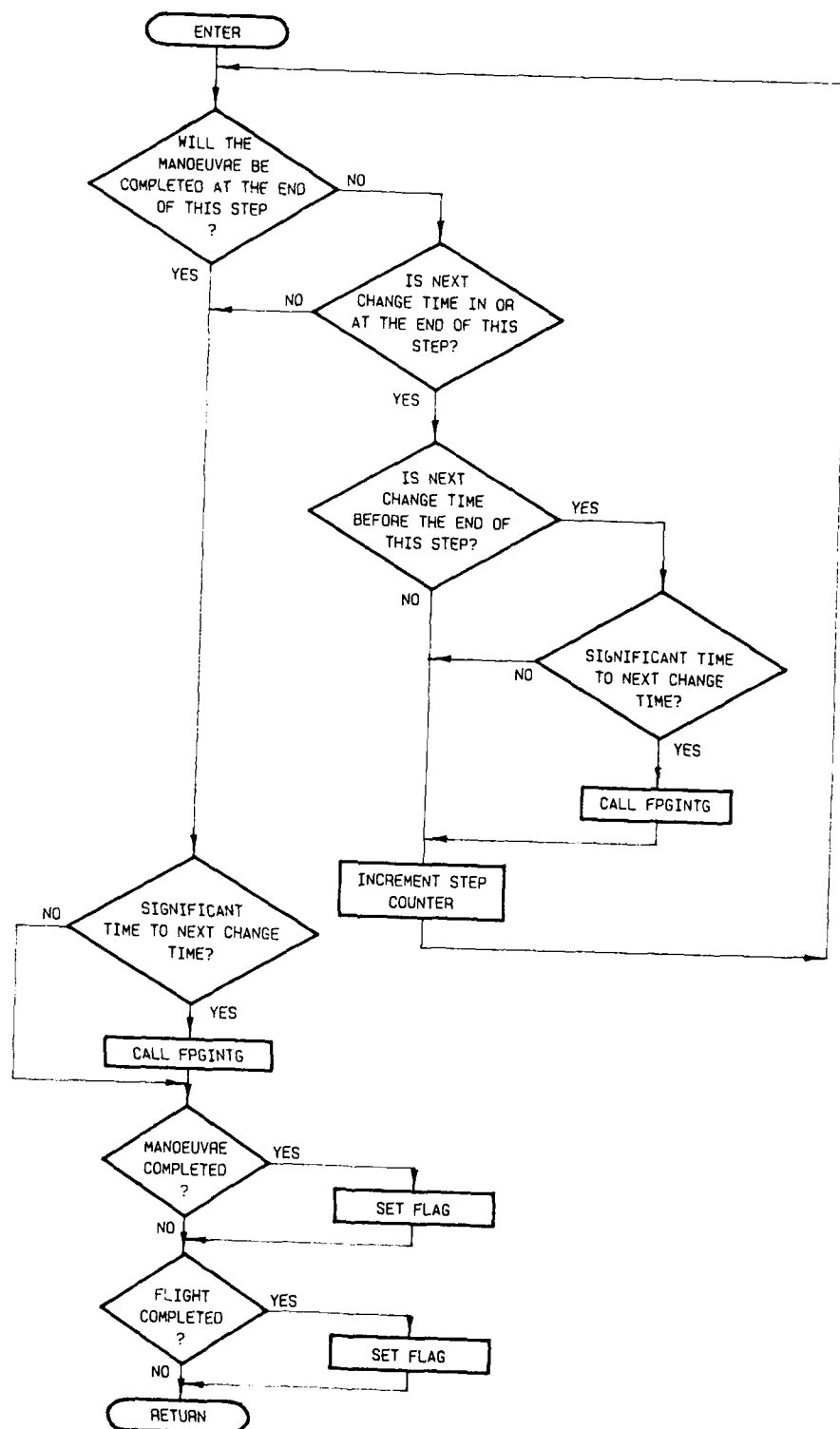
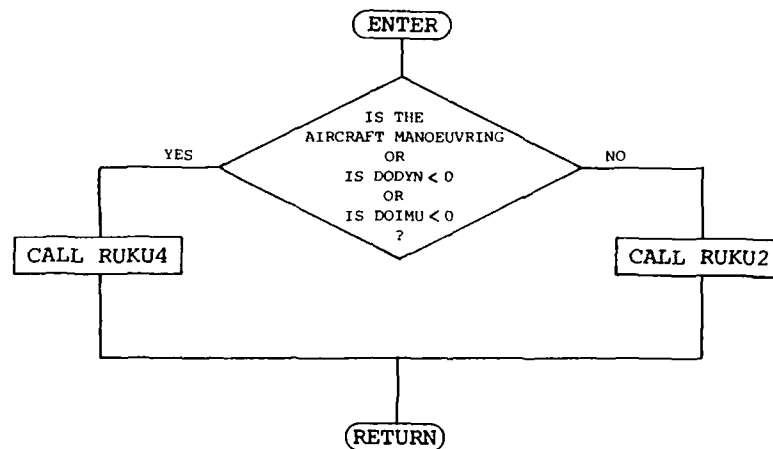


FIG. 4. FPGSTEP



The Runge-Kutta routines call COMVAR, which computes variables using the State Vector values.

These include:

- Direction Cosine Matrices between frames
- Velocity Components
- Altitude
- Latitude
- Local Earth Radii
- Local Gravity Components
- Angular Velocity Components (Nav. Axes):
  - Body to Nav from manoeuvre specifications
  - Nav to Earth from velocities
  - Earth to Intertial
- Angular Velocity Components (Body Axes):
  - Body to Intertial
- Linear Acceleration (Nav Axes)
- Specific Force (Nav Axes, then Body Axes)

The Runge-Kutta routines also call a routine FF, which returns derivatives of the state vector components as described in Section 4.1, using the results of the COMVAR calculations.

Figure 5 FPGINTG

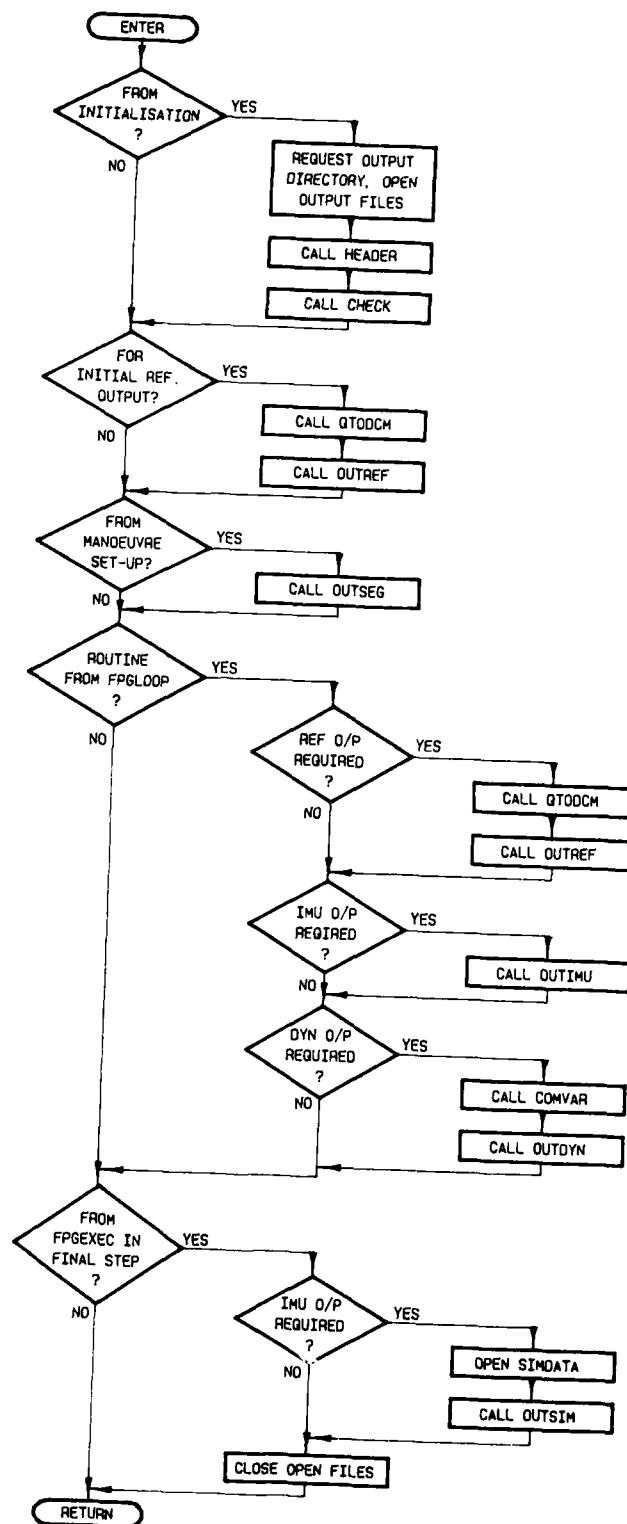


FIG. 6. FPGOUTS

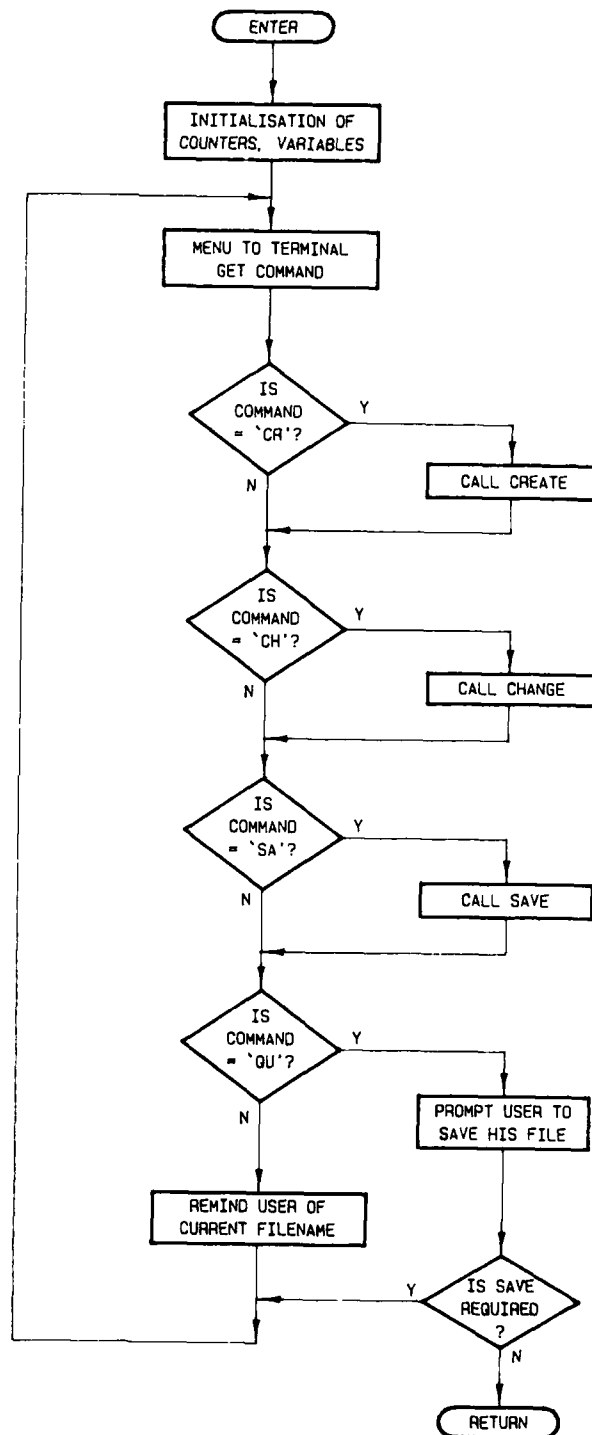


FIG. 7. WINDATA

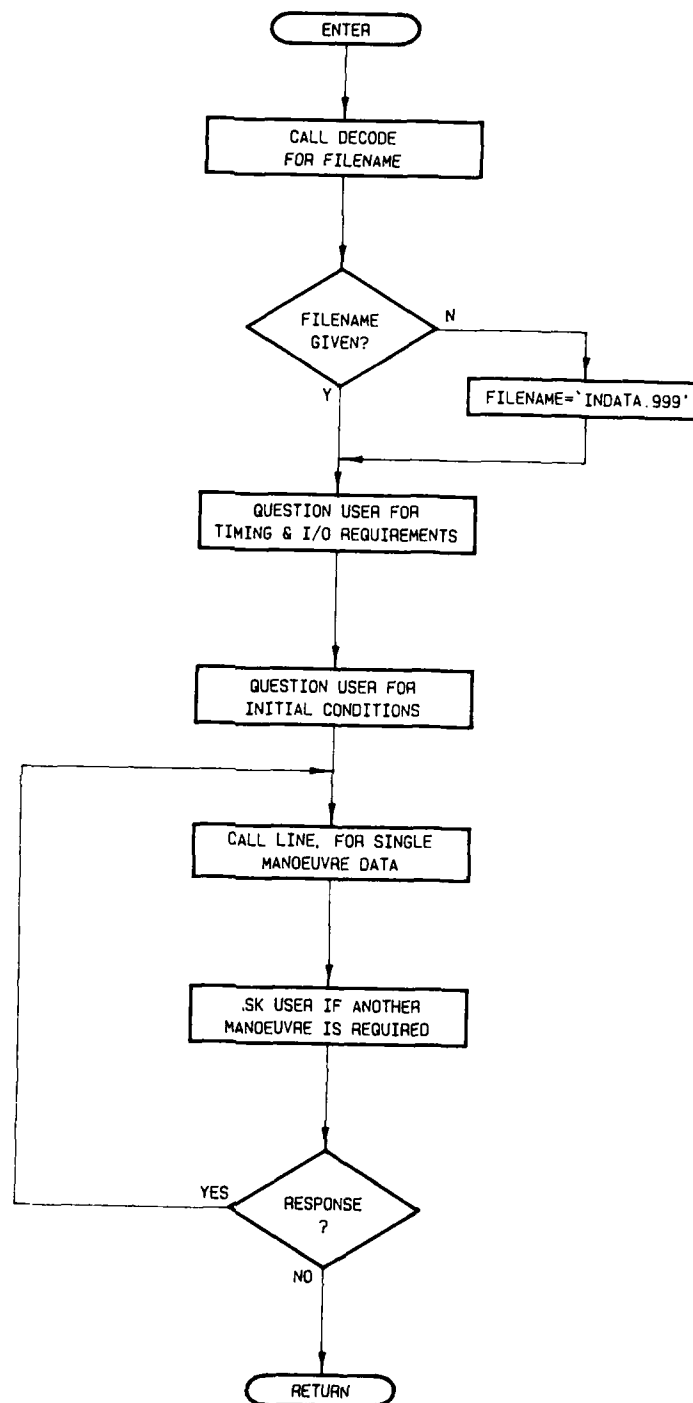


FIG. 8. CREATE

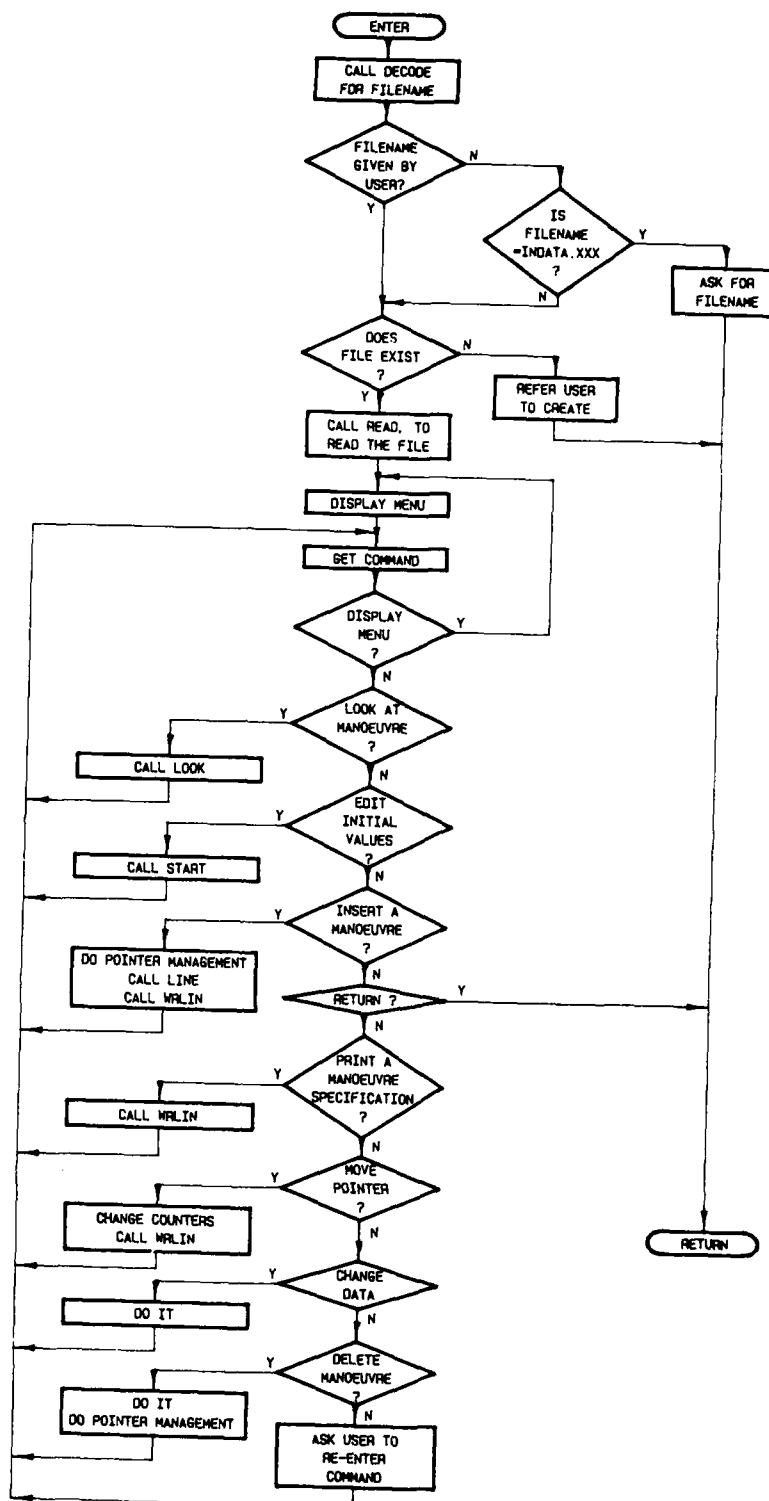


FIG. 9. CHANGE

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PAGE CLASSIFICATION  
UNCLASSIFIED

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1a. AR NUMBER AR-004-574	1b. ESTABLISHMENT NUMBER ARL-SYS-TM-98	2. DOCUMENT DATE JULY 1989	3. TASK NUMBER DST 86/064
4. TITLE FPG2 - A FLIGHT PROFILE GENERATOR PROGRAM		5. SECURITY CLASSIFICATION (PLACE APPROPRIATE CLASSIFICATION IN BOX(S) IE. SECRET (S), CONF.(C) RESTRICTED (R), UNCLASSIFIED (U) ).	
		<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">U</div> <div style="border: 1px solid black; padding: 2px;">U</div> <div style="border: 1px solid black; padding: 2px;">U</div> </div> <div style="display: flex; justify-content: space-around; font-size: small;"> <span>DOCUMENT</span> <span>TITLE</span> <span>ABSTRACT</span> </div>	
6. NO. PAGES 44		7. NO. REFS. 4	
8. AUTHOR(S) R.B. Miller		9. DOWNGRADING/DELIMITING INSTRUCTIONS Not applicable	
10. CORPORATE AUTHOR AND ADDRESS  AERONAUTICAL RESEARCH LABORATORY P.O. BOX 4331, MELBOURNE VIC 3001		11. OFFICE/POSITION RESPONSIBLE FOR:  SPONSOR _____ DSTO SECURITY _____ - DOWNGRADING _____ - APPROVAL _____ CSYD	
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14. DESCRIPTORS  Inertial navigation Strapdown navigational systems Flight simulation Computer programs		15. DRDA SUBJECT CATEGORIES  0076D	
16. ABSTRACT      This program simulates the environment of a strapdown inertial measurement unit in an aircraft executing a user-specified series of idealised manoeuvres. The program generates a file containing a sequence of specific forces and angular velocities in body axes coordinates, or a file containing a sequence of integrated (in body axes) specific forces and angular velocities, or both. The sequences are time-tagged, and also include aircraft height. For "reference" purposes it generates a file containing a sequence of nominally true position, velocity, and attitude of the aircraft.			

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16. ABSTRACT (CONT.)

17. IMPRINT

AERONAUTICAL RESEARCH LABORATORY, MELBOURNE

18. DOCUMENT SERIES AND NUMBER

Aircraft Systems  
Technical Memorandum 98

19. COST CODE

725326

20. TYPE OF REPORT AND PERIOD  
COVERED

21. COMPUTER PROGRAMS USED

22. ESTABLISHMENT FILE REF.(S)

G5/785

23. ADDITIONAL INFORMATION (AS REQUIRED)